1972

Estimating performance in skidding timber

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SMITH, Victor Gordon, 1927-
ESTIMATING PERFORMANCE IN SKIDDING TIMBER.

Iowa State University, Ph.D., 1972
Agriculture, forestry & wildlife

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A Dissertation Submitted to the

Graduate Faculty in Partial Fulfillment of

The Requirements for the Degree of

DOCTOR OF PHILOSOPHY

Major Subject: Forestry

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I. INTRODUCTION

Ways of estimating the productivity of timber-harvesting operations as exemplified by the timber-skidding process are investigated in this study. Investigations are made to better understand the relationships that exist between performance and related factors and to make recommendations to foresters and forest engineers on how to choose the estimation method that will give the most accurate estimate for fixed cost.

Forest managers use performance estimates in combination with hourly cost rates to determine the expected costs of various harvesting alternatives. Since hourly cost rates are relatively stable for one or two year periods while performance varies from one operating area to another, most short run differences in harvesting costs can be attributed to differences in performance rates. Thus, good estimates of performance are essential for the cost estimates used in selecting the proper harvesting method and for setting fair contract prices. Errors in estimating harvesting costs can be costly to forest managers and others.

The procedure adopted here was first to review current methods and research in performance estimation and then, using regression analyses, investigate the properties of a skidding operation in order to determine the best form of the performance estimator and to gain some insight into the relative advantages of using time-study or gross-data gathering techniques. Finally, the contributions to bias and variance due to sample size, non-linearity, and various commonly used approximation procedures for estimating values of the independent variables are examined using Monte Carlo techniques. Recommendations are then made based on these recommendations.

The analyses were made using large and medium wheel-skidder data obtained from the Forest Products Laboratory of the Canadian Department of Fisheries and Forestry. These data were gathered on the operations of three companies in eastern Canada during the summer of 1962. While the basic skidding technique has since been modified with the introduction of more powerful machines and different load assembly techniques, it is believed that the underlying relationships between performance, load size and distance have remained essentially the same, and that the estimation procedures described in this study are equally applicable not only to today's skidding operations in eastern Canada but to other timber harvesting processes in other geographic locations as well.

II. SHORT RUN HARVESTING DECISIONS

Timber harvesting is the processing and transporting of timber products from the growing site to the final delivery point. Harvesting costs refer to the dollar investment in labor and capital needed for timber harvesting.

The major group interested in estimating harvesting costs include woodland managers, logging contractors, company foremen and their staff and assistants. While their motivation may not be exactly that of profit maximization (Donnelly, 1964), the correspondence is close enough that the economic definitions of decision making and profit maximization are assumed to apply. The timber harvester maximizes profits by minimizing harvesting costs subject to constraints imposed by the firm and other institutions. He minimizes harvesting costs by selecting the harvesting alternative with the minimum expected cost for the anticipated set of environmental and operational conditions. To do this the decision maker must first identify and decide how to implement each alternative most effectively and then determine for each method the expected minimum cost.

Time limits the number of harvesting alternatives that a decision maker can include in his choice set. Over long periods of time, the decision maker can completely renovate and alter his inventory of harvesting equipment, as well as alter his forest holdings, so that he has almost complete flexibility. Over shorter periods of time, his choice set will be restricted by the stands he can harvest and by the equipment and labor force already at his disposal. Still, he has the option of using various road and landing patterns as well as various labor and equipment combinations in his different forest stands. In the very short run, on a day to day basis for example, the decision maker can implement only minor changes in equipment and labor allocations.

It is customary for the long run decisions to be made by top levels of management. Long run decisions pertaining to harvesting are usually made infrequently and require additional information about such things as manufacturing costs, and other long term supply and demand factors.

The short run decisions are made by middle management, e.g., at the "district" level. Cost estimates for short run decision making is emphasized in this study because these estimates are needed much more frequently than estimates for long run decisions.

There are two kinds of costs associated with performance estimation. There are the direct costs of data gathering and compilation in making the estimate. These costs are related directly to the number of observations made (sample size), and the duration of each observation (sample unit size) as well as the measurement technique, travel and fixed costs of survey. The second type of cost associated with performance estimation is the cost incurred due to error in the estimate.

If performance is under-estimated, estimated costs will be too high, and cost will be incurred in the form of unallocated or idle resources. If performance is over-estimated, estimated costs will be too low and the decision maker may suffer direct monetary loss, especially if he is a contractor. While theory has been developed expressing these costs directly by means of loss functions (Mood and Graybill, 1963), the more usual practice, and the one adopted in this study, is to use the standard error of estimate as a relative measure of the expected loss associated with a given estimate. The standard error in turn is inversely related to the square root of sample size; however, its relationship to the size of the sample unit is not usually specified, although relationships have been estimated by Freese (1961) and O'Regan and Arvanitis (1966) for area sampling situations.

Bias can result from the use of incorrect sampling methods, wrong prediction equations, or improper estimates of the independent variables used in the prediction equations. Although bias can be estimated and compensated for in some estimates, its influence is usually difficult to assess and only a limited number of correction procedures have been developed such as those described by Cochran (1963). One such method is the use of interpenetrating subsamples as suggested by Murthy (1963) to estimate the bias where two or more technicians collect and compile data.

III. CURRENT AND PROPOSED ESTIMATION PROCEDURES

The usual procedure for estimating short-run harvesting costs is to estimate the cost of each element separately and to sum the estimates to obtain a total cost; this is the method used by engineering economists (Matthews, 1942; Grant, 1950). This procedure for predicting total harvesting costs is extremely flexible yet simple. The basic assumption made is that each element is influenced by any other element by a fixed amount, and that delay costs caused by interaction among the various work elements is constant, regardless of the kinds of elements that are combined or the element being estimated. Although Matthews (1942), Lussier (1961a) and Corcoran (1964) have indicated that such an assumption could lead to estimates of suboptimal performance, some agencies do follow this practice (U. S. Forest Service, 1965; U. S. Department of the Interior, 1964; Hedbring and Akesson, 1966). Further study of the real importance of inter-element interactions seems indicated.

In most instances costs are found for each harvesting element by estimating a performance rate and multiplying this rate by an operating cost rate. The operating cost rate is usually expressed by hourly wage and machine rates, determined by standard accounting and engineering economics procedures (Grant, 1950). Since the same operating cost rates are also used by the accountants to determine actual harvesting costs, the difference between predicted and actual harvesting costs must originate from differences between estimated and actual performance rates. Estimation of element performance rates is therefore particularly important.

A variety of performance estimators are used by various decision-makers for different time spans (Tables 1, 2 and 3). Table 1 summarizes examples where future average performance is estimated by the arithmetic mean, based on the entire current season's performance or on a large sample from the current season.

Other estimators estimate future performance by modifying the current season's actual performance using ratios or regression coefficients (Table 2). These procedures utilize estimates of concomitant variables in the present and future operations and the relationship between these variables and current performance to predict future performance.

Still other agencies predict future performance by modifying an estimate of the current season's performance using relationships between performance and concomitant variables (Table 3). These relationships are determined from samples taken from the current season's performances and applied using predicted future values of the concomitant variables.

Implicit or explicit in the use of all these estimators is the assumption that current and future performances are parameters from some super population that could feasibly be estimated within the current and following years.

Two sampling rules are commonly used to collect data. The time-study method (Barnes, 1963; Killander, 1960) requires observations to be made on a continuous series of machine turns or cycles. With this method, the sampling unit is the turn.

The second method is the gross-data technique (Cottell and Winer, 1969; Worley _et al._, 1965) where observations are made in aggregated form over fixed periods of time such as the day or week. If longer periods of time are used, such as a week or month, the technique may also be referred

Table 1. Harvesting element performance estimated by unadjusted past season experience.

Source	Use of Estimate	Work Element
Matthews (1942)	Timber harvesting, short run	Felling log making loading
Gardner (1966)	Timber harvesting, long run	Felling skidding loading hauling
Corcoran (1964)	Transportation scheduling, long run	Skidding hauling
Tritch, Webster and Bentley (1968)	Regional harvesting cost analysis, long run	Felling skidding loading hauling
Industrial Experience - V. G. Smith	Cost estimation, short run	Felling skidding loading hauling

to as the accounting data method. With this method the sampling unit is a fixed period of time.

Each method has advantages and disadvantages. The time-study method is usually expensive since it requires continuous surveillance of the operation by the survey crew; however, a large number of observations can be accumulated in a short period of time. The gross-data method takes longer to collect data, but observations can be made at low cost by personnel regularly employed in other activities.

Sample size is usually determined on the basis of funds available rather than on precision criteria. Exceptions are the Battelle study and

Table 2. Harvesting element performance estimated by adjusted past season's performance.

Source	Use of Estimate	Work Element	Adjustment Method
U. S. Forest Service (1965)	Timber appraisal, short run	Fell and buck prehaul load	Ratios based on time studies. To correct past season costs.
Donnelly (1964)	Annual mgmt. plan, short run	Prehaul	Multiple regression using gross data.
Hilliker, Webster and Tritch (1969)	Regional cost analysis, long run	Felling prehauling loading hauling	Simple linear regression, using accounting data.
Pulp and Paper Research Institute of Canada (Bennett et al., 1965)	Environmental factor analysis, short run	Prehaul	Multiple regression with gross data.

the ratio-delay method. In the Battelle study (Hamilton et al., 1961) the necessary sample size was determined by estimating the sample variance at intervals as the number of sample units was increased until the sample variance had reached a prescribed level.

Many of the currently used estimators (Tables 1, 2 and 3) were developed by multiple regression analyses using series of observations from time studies. However, Donnelly (1964) and Bennett and Winer (1967) have indicated that successive observations are likely autocorrelated and that the independent variables selected are often measured with sampling error and that they are multicollinear. The joint result of these several complications cannot be inferred as the sum of their separate

Table 3. Harvesting element performance estimated by sampling from previous performance.

Source	Use of Estimate	Work Element	Estimate Method
Bureau of Land Mgmt. (U.S. Dept. of Interior, 1964)	Timber appraisal, short run	Fell, buck, pre-haul, loading, yarding.	Multiple regression with time study
Battelle Study (Hamilton et al., 1961)	Regional analysis, long run	Fell, buck and load	Multiple regression with time study
Can. For. Prod. Lab. (McCraw, 1967)	Environmental factor analysis, short run	Prehaul	Multiple regression with time study
Skogsarbeten, Sweden (Hedbring & Akesson, 1966)	Equipment design, long run	Fell, prehaul, load, slash	Ratios based on time study
For. Engr. Lab. Morgantown (Wren, 1962)	Cost estimation, short run	Fell, prehaul	Multiple regression with time study
Can. For. Mgmt. Inst. (Newnham, 1967)	Equipment design, long run	Prehaul	Ratios based on time study
U.S. Forest Service (Merz et al., 1965)	Cost estimation, short run	Felling	Multiple regression with timber cruise and time study
Lussier (1961a)	Cost control, very short run	Fell, prehaul, load, haul	Ratio delay and gross data
U.S.F.S. Rocky Mtn. Sta. (Schillings, 1969)	Cost estimation, short run	Prehaul	Ratios and multiple regression using time study

results (Johnston, 1963, p. 216), and Donnelly (1964) suggests that estimates obtained under these conditions should be used with caution.

Generally the predictive properties of these equations are determined by the data collection method and by the characteristics of the performance population.

Many investigations have sought the important independent variables that affect wheeled skidding; for example, McCraw (1967), Winer (1967), Cottell and Winer (1969), Wren (1962), Donnelly (1964) and Schillings (1969).

The general conclusion summarized by Winer (1967) is that variables directly associated with load size and travel time are most important and that these variables may be either controlled variables such as skidding distance and crew size, or uncontrolled variables such as volume per tree and volume per acre. Environmental factors such as brush, windfall, terrain, species, etc., are usually weakly related to skidder productivity.

All the studies include load size, as measured by log or tree volume or number of logs, as an important variable. One-way skidding distance was second in importance. Donnelly (1964) discarded skidding distance "on statistical grounds". He rationalized this decision on the basis of high flexibility in crew organization and the high speed of wheeled skidders which combined to make distance less important. He also felt that crew aggressiveness tended to counteract the effect of variations in skidding distance.

Both Winer (1967) and Donnelly (1964) considered crew aggressiveness to be a factor of prime importance, although Winer noted the difficulty of predicting this factor on a seasonal or annual basis.

Wren (1962) and Schillings (1969), who conducted their studies in mountainous regions, included slope as an important factor, but in the hilly to flat regions of eastern Canada where McCraw and Winer conducted their studies, slope was not found important.

Similarly, trail preparation was found by McCraw to be important but Winer found it only of importance in predicting travel time.

Some grouping also seems necessary. The load sizes and travel speeds, acceleration rates, and maneuverability of the various prehauling units would indicate different functional relationships between performance and load size and distance for different machine-types. Separate prediction equations for each company using the same equipment may also be necessary since different companies harvest different types of forest products and use different management practices, labor organization, and incentive schemes. Hence Winer (1967) notes a reluctance among companies to use prediction equations based on industry-wide data. There may be no need to develop different equations to predict performance in different forests stands on a single company's operations since many of these stand characteristics are reflected in the average skidding distance, average tree volume and number of trees per load used as concomitant variables in the prediction equations. This is a question for investigation.

Season has a pronounced influence on prehauling performance. In the summer, loading is faster, and traveling slower because of soft, wet spots that develop in the skidding trails and because of the drag of sand on the load. In the winter, loading is slower because of the snow and because tree-lengths become frozen to the ground, but travel time is faster because of improved trail conditions.

Delays are also related to performance in a fairly complex and profound way. As in other industrial situations, prehauling delays occur at random points in time. Thus, longer cycle times will include a larger delay-time component.

The ratio-delay method (Lussier, 1961b) is sometimes proposed for performance estimation. This is a method of analysing harvesting elements by making observations at random points in time. The objective of this technique is to estimate the proportion of time spent on various phases of a harvesting element in order to identify inefficiencies in the operation. This technique is not used to supply data directly for short-run performance estimation unless it is coupled with measurements of productivity over some specified time period.

Gardner and Schillings (1969) have studied the efficiency of ratio-delay and time-lapse photography as compared to continuous time study. On some operations, data can be collected economically using a random-sampling procedure to select time units or turns for observation. The distinction between using randomly selected units and units in a series is especially important if the units are autocorrelated. Sample units have been customarily observed in series by most agencies who must incur high travel and idle-time costs if other intermittent observation methods were used.

IV. NOTATION

The following notation is used in the subsequent chapters and in the appendices.

b_o	Regression coefficient denoting intercept term.
b_i	$i = 1, 2, 3, \ldots, k$. Regression coefficient.
b_w	Regression coefficient estimated from within day sums of squares and cross products.
D_{ij}	$i = 1, 2, \ldots, N$; $j = 1, 2, 3, 4$. Skidding distance associated with the i^{th} observation using the j^{th} type observation unit. $j = 1$ when machine turns are observed. $j = 2$ when performance over fixed periods of time are observed (e.g., day, week). $j = 3$ when uniform volumes of production (e.g., cord, cunit) are observed. $j = 4$ when load sizes are observed.
D^*	A given or specified skidding distance.
$\bar{D}_j$	$j = 1, 2, 3, 4$. Mean skidding distance for the j^{th} type unit.
$\hat{\bar{D}}$	Estimated mean skidding distance.
$E()$	Expected value of variable included in parentheses.
L_{ij}	$i = 1, 2, \ldots, N$; $j = 1, 2, 3, 4$. Load size associated with the i^{th} observation of the j^{th} type observation unit.
L^*	Given or specified load size.
$\bar{L}_j$	$j = 1, 2, 3, 4$. Mean load size for j^{th} type observation unit.

$\hat{L}$	Estimated mean load size.
N_j	j = 1, 2, 3, 4. Number of j^{th} type units in the population.
n	Sample size (usually total number of turns).
n'	Number of days in which samples are taken.
n_i	Number of turns observed in the i^{th} day.
P()	Probability associated with value of variable included in parentheses.
P	Performance (minutes/cunit).
$\hat{P}$	Estimated performance.
$\overline{P}^{-1}$	Average performance (cunits/minute).
$S^2_{\hat{P}}$	Sample variance for estimated performance.
$S^2_{y.x}$	Residual mean square error.
$S^2_{y.x_b}$	Residual mean square error for a between-days sums of squares and cross products estimate.
$S^2_{y.x_w}$	Residual mean square error for a within-days regression estimate.
$S^2_{b_b}$	Error variance associated with a between-days estimate of a regression coefficient.
$S^2_{b_w}$	Error variance associated with a within-days estimate of a regression coefficient.
$S^2_{S^2_{\hat{P}}}$	Sample variance associated with an estimate of performance sample variance.
$SE_{\hat{P}}$	Standard error of estimated performance.
$SE_{y.x}$	Standard error about regression line.

SE_b Standard error associated with estimate of regression coefficient.

T_{ij} $i = 1, 2, \ldots, N$; $j = 1, 2, 3, 4$. Time associated with i^{th} observation unit of the j^{th} type observation unit.

$\bar{T}_j$ $j = 1, 2, 3, 4$. Mean time associated with j^{th} type of observation unit.

$\hat{\bar{T}}$ Estimated mean time.

$V()$ Variance of variable included in parenthesis.

$\sum_{i=1}^{N}$ Summation of variables with subscripts from $i = 1$ to $i = N$.

W_i Weighting factor associated with i^{th} observation (usually the inverse of variance).

$W_i = 1/S^2_{y.xi}$ for time study estimates.

$W_i = n_i$ number of turns/day for gross data estimates.

$[x^2]$ Sum of squared deviations of variable x about its mean ($x = D$, L or T).

$[Wx^2]$ Weighted sum of squared deviations of variable x about its weighted mean.

$[xy]$ Sum of cross products of deviations of variables x and y about their respective means ($x = D, L, T$; $y = D, L, T$ but $y \neq x$).

$[x^2_w]$ or $[xy_w]$ Within-group sums of squares or cross products.

$[x^2_b]$ or $[xy_b]$ Between-group sums of squares or cross products.

V. STUDIES INDICATED

The preceding sections have defined the decision making process and the role of performance estimation. Current and proposed prooedures for performance estimation were reviewed. As a result, questions can now be raised about performance estimation that will then be investigated.

First, what are the relationships between performance and the independent variables? These relationships should be more clearly defined. Other than identifying load size, distance and crew aggressiveness as well as machine-type and company as important factors, their relationship to performance remains to be defined. Also, further investigation of autocorrelation and heteroscedasticity in the prediction error term seems warranted. Under what circumstances is the time-study technique superior to the gross-data technique, and when is it inferior? If the relationship between performance and load-size, distance and other factors varies when the data is aggregated, the estimates from these two procedures may be different. Thus the heterogeneity properties of the data should be investigated. These investigations should include tests of homogeneity of variance to see if weighted regressions should be used; model fitting should be done to find the best functional form of the relationship for predicting performance, and the regression coefficients for various groups of data must be compared to see if some or all of the coefficients are constant among these sets of data. From these investigations, inferences can be made about the generality of the various regression relationships.

A second set of questions pertain to estimating the precision of performance estimates when time-study and gross-data gathering techniques are used. For time-study and gross-data methods how many observations must be made to provide an estimate of specified precision?

What are the contributions to bias of various estimation procedures commonly in use? These procedures include the use of an estimate of average skidding distance squared ($\hat{\bar{D}}^2$) to estimate the expected value of distance squared, $E(D^2)$, the use of the reciprocal of estimated load size ($1/\hat{\bar{L}}$) to estimate the expected value of the reciprocal of load size, $E(1/L)$, and finally the use of the reciprocal of performance ($1/\hat{P}^{-1}$) where performance is expressed in terms of cunits per minute ($\hat{P}^{-1} = V/T$), to estimate performance (P), when performance is to be expressed in terms of minutes per cunit ($P = T/V$).

Time and data restrictions prohibit the investigation of other performance prediction problems. For example, what is the effect of interplay among various combinations of harvesting elements and how do seasonal changes affect performance?

The investigations that were made could have been carried out on any one of the elements of a harvesting operation; however, data were most readily available for the timber skidding work element. Timber skidding best exemplifies the problems of performance estimation in a forest environment and much work has already been done in developing estimators for this element. We shall assume that the concepts investigated here will be equally applicable to the other harvesting elements such as felling, bucking, slashing, hauling, etc. The ideal would be to examine directly the properties of the other harvesting elements; however, there is now neither time, money nor data for such an extensive investigation.

The specific prehauling method investigated is the tree-length, wheeled-skidder method used by three eastern Canadian pulp and paper and lumber companies using "large" and "medium" powered wheel skidders.

These operations were carried out with four-wheel drive vehicles powered by 60 to 110 hp engines and equipped with winches and A-frames. Loads consisting of from three to ten tree-lengths were assembled and skidded to a central landing. Skidding distances ranged from 0 to 3,500 ft. or more. In addition to the skidding operations, some skidding units were used briefly for skid-trail construction and for constructing cold decks at the landing. Many of the main skid-trails were roughly bulldozed to permit faster travel speeds. Crews were paid either on an hourly basis or on an hourly basis plus bonus.

The data were gathered during the summer of 1962 using methods described by McCraw (1962, 1964). Turn characteristics for from two to five machines were observed and recorded concurrently and it was impossible later to consistently distinguish between the performance of one machine and another from the data alone. While considerably more turn data were gathered in the stands harvested, the following data were considered relevant to these studies:

-- load size - cunits (1 cunit = 100 cu. ft. solid wood)

-- one way skidding distance - 100's of feet

-- gross turn time - 1/10 of minutes

-- company identification number

-- date

-- forest cover type

-- machine type - "large" - greater than 100 hp

"medium" - between 50 and 100 hp

A total of 614 turns were observed and their distribution by days, company and machine type is shown in Table 4.

Table 4. Summary of data studied.

Day No.	Co. No.	Machine Type[a]	No. Obser-vations	Average Performance (min./cunit)	Ave. Load (cunits)	Ave. Dist. (feet)
1	1	LWS	28	10.54	1.99	1098
2	1	LWS	47	10.10	1.82	1143
3	1	LWS	11	8.89	1.91	338
4	1	LWS	34	8.05	2.16	1017
5	1	MWS	19	31.27	.57	442
6	1	LWS	18	11.29	1.83	1294
7	1	MWS	17	26.74	.62	429
8	2	MWS	89	20.35	.82	1172
9	2	MWS	70	17.95	.68	1673
10	2	MWS	40	24.88	.69	1418
11	2	MWS	55	35.96	.72	1264
12	2	LWS	23	27.99	1.35	2742
13	2	MWS	6	43.73	.78	2880
14	3	MWS	23	29.93	1.48	3380
15	3	LWS	61	21.37	1.19	1712
16	3	LWS	73	10.80	2.08	2144
Total			614	16.71		

[a]LWS - Large wheel skidder, 100 horsepower or greater; MWS - Medium wheel skidder, 50-100 horsepower.

VI. SELECTION OF A PERFORMANCE EQUATION

A. Introduction

The performance relationships typically encountered in skidding timber are investigated in this chapter. After defining the dependent and concomitant variables and defining the observation units used for estimation, regression analyses are described that were done to identify a suitable equation and to examine the heterogeneous properties of the populations. In addition to identifying an estimator and some of the population properties, these investigations give an insight into the relationship between time-study and gross-data survey techniques.

B. Dependent Variable

The dependent variable is determined by the decision-maker's information need, which is an estimate of the total cost of harvesting a fixed quantity of timber. The decision-maker estimates this total cost by multiplying the per cunit cost of harvesting by the total volume of wood to be cut. The per cunit harvesting cost is the sum of the per cunit element costs. Thus, for the skidding element, the decision-maker wants an estimate of the cost per cunit. This is the product of the skidding performance rate and the cost rate for operating a skidding unit. Since the cost rate is measured in dollars per time-unit, the performance rate must be measured in terms of time-units per cunit production, where the time-unit is usually expressed in hours or minutes. The dependent variable is therefore the performance rate measured in minutes per cunit ($P = T/L$).

C. Concomitant Variables

The concomitant variables and measurement units used in these investigations have been determined partly by the data that were available and partly by the conclusions noted in the skidding studies described in Chapter III. As a consequence, load-size, measured in hundredths of cunits, and skidding distance, measured in hundreds of feet, were the independent variables. The effect of machine-type, company and day differences were examined in various groups of data. The average effects of stand, crew aggressiveness and delays on performance were assumed to be described by the intercept terms in the prediction equations. Crew aggressiveness could not be treated as an independent variable because no measure of this factor was included in the data.

D. Populations Considered

The parameter to be estimated is performance (P) measured in terms of minutes per cunit production,

$$P = \sum_{i=1}^{N} T_i / \sum_{i=1}^{N} L_i$$

where

$\sum_{i=1}^{N} T_i$ is the sum of future times required, (in minutes)

$\sum_{i=1}^{N} L_i$ is the sum of future volumes skidded (in cunits), and summation is over all the sampling units (N) in the future operation.

The population of interest may be viewed as made up as either one of four kinds of sampling units. First, the population may be defined as a collection of round-trips or machine-turns. With the turn as the population unit, the performance parameter for the population (P) is the sum of times associated with each turn ($\overset{N}{\Sigma} T_{1i}$) divided by the total load volumes skidded on each turn ($\overset{N}{\Sigma} L_{1i}$). Turns are the observation units commonly used in time-studies. Performance in the turn population can therefore be expressed as:

$$P = \left(\sum_{i=1}^{N_1} T_{1i}\right) / \left(\sum_{i=1}^{N_1} L_{1i}\right)$$

Analogously, the performance ratio (P) can be expressed as average turn-time divided by the average load-size, since

$$P = \left(\sum_{i=1}^{N_1} T_{1i}/N_1\right) / \left(\sum_{i=1}^{N_1} L_{1i}/N_1\right)$$

$$= \overline{T}_1/\overline{L}_1$$

The performance ratio may also be looked upon as being a parameter of a population of uniform time intervals such as days, weeks or hours. Hence, performance (P) would be the sum of the fixed time intervals ($\overset{N}{\Sigma} T_{2i}$) divided by the sum of the volumes skidded in each time interval ($\overset{N}{\Sigma} L_{2i}$). The daily interval is the observation unit used in gross-data surveys. An equivalent expression of the performance ratio (P) is:

$$P = \left(\sum_{i=1}^{N_2} T_{2i}/N_2\right) / \left(\sum_{i=1}^{N_2} L_{2i}/N_2\right)$$

$$= \overline{T}_2/\overline{L}_2$$

Therefore, if $\overline{T}_2$ is equal to a day then P is the "inverse of the average volume skidded per day" $(1/\overline{L}_2)$.

Third, the performance ratio for the future skidding operation may be viewed as a parameter of a population of uniform production (volume output) units such as cunits or cords. Here the performance ratio (P) would be the sum of the skidding times associated with each production unit $(\Sigma^{N} T_{3i})$, divided by the total volume of production $(\Sigma^{N} L_{3i})$. Analogously, performance (P) can be expressed as the average time required per specified unit of production,

$$P = \left(\sum_{i=1}^{N_3} T_{3i}/N_3\right)/\left(\sum_{i=1}^{N_3} L_{3i}/N_3\right)$$

$$= \overline{T}_3/\overline{L}_3$$

Hence, if $\overline{L}_3$ is one cunit, then $\overline{T}_3$ is the average time required to skid a cunit of wood.

Finally, the performance ratio may be considered a parameter of a population of future loads. Here performance (P) is found by summing the product of the performance ratio by the ratio of load size to total volume for each load.

$$P = \Sigma^{N_4} (T_i/L_i) \ (L_i/\Sigma^{N_4} L_i)$$

The future performance parameter is not observable in the currently available portions of the populations just described, i.e., it is not possible to sample future performance. However, past performance is related to some observable variables which can be estimated in the future. Therefore the future performance parameter must be estimated indirectly in the following manner.

For performance (P) based on the turn as the sampling unit, average time per turn ($\overline{T}_1$) must be estimated using the relationship between turn-time, load-size and distance that is observable in the present operation with predicted values of load-size and distance determined for the future operating area. Future load-size and distance are "controlled" variables, specified in part by the number of tree-lengths per load and by the landing interval respectively. To estimate load-size, the average tree size in the future operating area must also be estimated.

For performance based on uniform time intervals as the sampling unit, a common procedure is to estimate the average volume per time interval ($\overline{L}_2$) using the relationship between volume per time interval, load-size and distance as estimated in the currently observable part of the population and using predicted values for future load size and skidding distance.

For performance based on uniform production units as the sampling units, the average time per unit production ($\overline{T}_3$) must be estimated from the relationship between T_3 and load-size and distance as observed in the current population, and then applying this relationship with predicted values of load-size and distance for the future operation.

Some of the estimators of performance commonly used in the turn and uniform time populations will be biased since they include random variables in their denominators (i.e., $\hat{P} = \hat{\overline{T}}_1/\hat{\overline{L}}_1$, and $\hat{P} = \overline{T}_2/\hat{\overline{L}}_2$). This means that the reciprocals, $(1/\overline{L}_1)$ and $(1/\overline{L}_2)$, are being biasedly estimated by $(1/\hat{\overline{L}}_k)$, k=1,2, instead of $(\widehat{1/\overline{L}_k})$ (Murthy and Pillai, 1966).

The question of how much these estimators will be biased in usual performance estimation situations is investigated in Chapter VIII.

E. Model Fitting

Regression analysis was used to identify the most consistently "best" fitting model for the sixteen sets of daily data (Table 4).

Loads were used as sample units which permitted performance (minutes/cunit) to be estimated directly as the independent variable. Had turns or fixed time units been selected, an additional multiplication by the inverse of average load size would have been needed to estimate performance.

A full model (all independent variables included) as well as its various reduced forms was fitted to each set of data. Residual errors were then examined to determine what additional variables should be included. After examining the residual mean squares for the different models, a "best" model was selected (Appendix A).

The most satisfactory model for these sets of data was:

$$P = b_1(1/L) + b_2(D/L)$$

A considerable proportion of performance variation remained unexplained however, and variation existed among the performances for different days. Consequently, further analysis of these heterogeneous data was indicated.

F. Analysis of Heterogeneous Data

The analysis of heterogeneous data was done to determine whether the separate groups of daily data could be combined without unduly increasing the standard error of the equation due to "between-day" sources of variation (Appendix B). In this and all subsequent analyses, the model $P = (b_o + b_1 D)/L$ is used. This model is the one used in time-study work where turns are observed. The model exhibited virtually the same error properties as the performance equation described in Part E where loads were used as observation units. The data indicated differences between daily sets of performances, and while the relationship between turn-time and distance could be expressed by a common coefficient (b_1) for all company-machine groups using a weighted regression, the intercept term (b_o) should be estimated separately for each day, or for each company, machine-type combination.

Two methods of estimating the coefficient (b_1) were compared (Appendix B). The first "overall" estimate was

$$\hat{b}_{1o} = \sum_i^{n'} W_i \sum_j^{n_i} (x_{ij} - \bar{x}')(y_{ij} - \bar{y}') / (\sum_i^{n'} W_i \sum_j^{n_i} (x_{ij} - \bar{x}')^2)$$

using weighted sums of squares and cross-products of deviations from weighted means

$$\bar{x}' = \sum_i^{n'} W_i \sum_j^{n_i} x_{ij} / \sum_i^{n'} W_i n_i \quad \text{and} \quad \bar{y}' = \sum_i^{n'} W_i \sum_j^{n_i} x_{ij} / \sum_i^{n'} W_i n_i$$

where $\quad W_i = 1/S^2_{y.x_i}$

This is the standard weighted regression estimator.

The second "combined" estimate, found by summing appropriately weighted within-day sums of squares and crossproducts was

$$\hat{b}_{1c} = \sum_i^{n'} \sum_j^{n_i} W_i (x_{ij} - \bar{x}_i)(y_{ij} - \bar{y}_i) / \sum_i^{n'} \sum_j^{n_i} W_i (x_{ij} - \bar{x}_i)^2$$

where $\bar{x}_i = \sum_j^{n_i} x_{ij}/n_i$ and $\bar{y}_i = \sum_j^{n_i} y_{ij}/n_i$

and

where $W_i = 1/S^2_{y.x_i}$

The "overall" estimate $(\hat{b}_{1o})$ was generally more precise than the comparable "combined" estimate $(\hat{b}_{1c})$, even though the residual mean-squares are smaller for the combined estimates than for the overall estimates (Appendix B). This greater precision is directly attributable to the fact that the range of distances over which observations were made for all days was considerably greater than the range for any one day.

Should the "overall" estimate $(\hat{b}_{1o})$ be used since it is more precise? The answer is not obvious because if the overall estimator $(\hat{b}_{1o})$ is used, a larger residual mean-square might be obtained which could result in a poorer performance estimate. Therefore the precision of performance estimates using both estimators is further investigated in Chapter VII.

Within-day estimates of b_1 using either the "combined", or "overall" estimators are well adapted for use with the time-study data gathering technique where individual turns within a number of days are observed and recorded. The gross-data technique, on the other hand, uses aggregated daily sets of performances to estimate b_1. This is equivalent to using between-day sums of squares and crossproducts. Assuming that the daily aggregates are randomly ordered, the within- and between-day estimates were compared to give some insight into the efficiency of these two data gathering techniques.

The analyses of the efficiency of time-study and gross-data techniques for estimating the regression coefficient (b_1) indicated that both the within-day and between-day estimates were estimating the same relationship (b_1) between turn-time and distance (Appendix C). However, the within-day estimates were up to fifty times more efficient than the between-day estimates if sampling fractions (n/N) and costs were the same for both methods. The within-day estimates are possible only if time-study data is available. On the other hand, the between-day estimates can be based on either time-study data or gross-data.

We still cannot freely recommend the use of within-day estimators of the time-distance relationship as part of the procedure for estimating skidding performance. Instead, the relative precision of the within-day and between-day estimates of skidding performance must be further investigated. This is done in the next chapter.

VII. EFFICIENCY OF TIME-STUDY AND GROSS-DATA METHODS

A. Introduction

In the last chapter performance equations were selected that best fit the data available. Also, the efficiency of the time-study and gross-data techniques for estimating the time-distance relationship (b_1) was discussed. We also found that the time-distance relationship is relatively constant from day to day, while other unidentified factors caused important daily variations in performance. We will now see how this information can be used to obtain more precise performance estimates.

The precision of performance estimates is measured by estimates of sample variance. Variance estimators are available for estimates based on random samples; however, both the gross-data and time-study techniques usually produce serial observations because of cost considerations. Therefore the relative precision of the two estimation methods can be examined, only after we know something about the properties of random sample variance estimators when used with such serial observations as these.

B. Properties of Performance and Variance Estimators in Serial Samples

Serial sampling properties resemble the properties of either systematic or one-stage cluster sampling. As Cochran (1963) has indicated for systematic sampling, if the population were randomly ordered, i.e., no important day-to-day differences occur, or if no trends in daily performance exist, then an estimate made with a serial sample would have the same bias and variance properties as estimates made from a random sample. Insufficient data exist to determine whether daily performances are indeed random, therefore, we cannot really tell whether random sample variance estimators satisfactorily estimate the variance of gross-data estimates or not.

The variance of time-study estimates can be investigated with more certainty however. If no important day-to-day or other systematic variations in performance exist, then the random sample variance estimator should provide a good estimate of sample variance for time-study estimates. If the population were heterogeneous and all variation occurred between days and none within days, the result of using a random sampling variance estimator with a serial sample can be illustrated empirically as shown in Table 5 using fictitious data. In this example the random sample variance estimator was calculated for all possible serial couples and the expected values of the estimated mean, variance and sample variance were calculated. It can be seen that the expected value of the sample variance (.167) is less than the parameter estimated (.367). There are two apparent sources of this error. First, the estimate of the population variance using serial samples is negatively biased (i.e., .50 c.f. .67). Second, even if the true population variance had been unbiasedly estimated, the random sample variance estimator can only give unbiased estimates of cluster sample estimates if intercluster correlation is zero (Cochran 1963, p. 242). In this example the correlation coefficient has a value of .545 which means the random sample variance estimator is negatively biased.

Table 5. Serial sampling in a heterogeneous population.

Day No.	Performance Rate	Sample Units Drawn	Estimated Mean Performance	Estimated Population Variance	Estimated Sample Variance
1	2	2,3	2.5	.5	.167
2	3,3	3,3	3.0	0	0
3	4,4,4	3,4	3.5	.5	.167
		4,4	4.0	0	0
		4,4	4.0	0	0
		4,2	3.0	2.0	.667
Total	20		20.0	3.0	1.000
Average	3.3		3.3	.50	.167

True mean performance - 3.3 Expected value of estimated performance - 3.3

True sample variance - .367 Expected value of estimated variance - .167

True population variance - .67 Intercluster correlation coefficient - .545

This example suggests likely sources of error in variance estimation. The example does not indicate the magnitude and the sign of the bias introduced by each source in actual skidding populations since the structure of such populations is quite different than the structure of the population used in this example.

As variation among daily performances is more nearly the result of within-day sources of variation, the variance estimates will tend to be less biased. Further, as the sample size is increased and samples become more representative, the estimates should also be less biased.

To illustrate these properties in more detail, Monte Carlo studies were conducted using the large-skidder performance data for days 1, 2, 3, 4, 6, 12, 15 and 16 as listed in Chapter V.

C. Monte Carlo Studies

The time-study estimators were investigated using Monte Carlo methods. Expected values of "combined" and "overall" estimates of performance and sample variance were estimated to determine the trends in bias and variance with changing sample size.

The Monte Carlo techniques used were similar to those used by Schreuder et al. (1968) and O'Regan and Palley (1965). The method has both advantages and disadvantages. The advantages include considerable savings in time and money compared to the use of "real world" studies, and the response to controlled changes in estimation procedures is unobscured by long term or other undefined trends that plague the users of "real" data. On the other hand, the technique has disadvantages. The population used for the studies is strictly a sample and may not adequately represent the true situation. Thus, misleading inferences could be made. In this particular study observation units have been pooled from the operations of three different companies, therefore, the trial population may be unrealistic and too small to properly portray the sampling properties of skidding operations in general.

Monte Carlo studies examine the bias and variance of an estimation procedure when used in a particular parent population. Estimates obtained from repeated samples are averaged, and the average is used to estimate the expected value of the estimates. The sample variance of the estimated expected value can be reduced by increasing the numbers of estimates drawn from the parent population, and by careful choice of the Monte Carlo sampling rule.

In the Monte Carlo study made here, skidding performance is estimated using n(n = 50, 75, 100, ..., 200) serial observations drawn from the parent population of 295 turns. Thus, 295 different sets of n serial observations were selected using a "looping" procedure. Looping is used if the first turn in the sample, i.e., the kth population unit, is greater than the 295-n population unit. The sample would then include all turns from k to 295 inclusive, and turns from one to n + k - 294 inclusive. The serial sample starting with the kth population unit will vary from its neighbors, starting with the (k - 1) and (k + 1) population units, by only the first and last units. Thus, estimates obtained from successive sets of serial observations may be expected to show smooth trends or changes. The Monte Carlo sampling rule used this characteristic of the sample population to reduce the variance of the estimated expected values. The 295 sets of n serial observations were divided into 40 approximately equal segments. Two sets of n serial observations were then drawn at random from each segment, and the 80 sets of observations thus obtained were used to calculate the Monte Carlo estimates of expected estimated performance and estimated sample variance. The stratified random sample variance estimator was used to estimate the sample variance for each Monte Carlo estimate.

The autocorrelation properties were retained by preserving the order of the turns in the sample population.

The mean load-size and mean skidding-distance for the large-skidder population were the values assigned to the independent variables (D* and L*) in all the estimates generated, and the parameter estimated was the current average performance of the large-skidder population (P). Thus, errors in performance estimation due to errors in estimating the future levels of the independent variables are not indicated by this procedure.

The performance estimator used in these investigations was

$$\hat{P} = (1/L^*)\ (b_0 + b_1 D^*)$$

The random sample variance estimator with the finite population correction factor was

$$\hat{S}^2_{\hat{P}} = (\hat{S}^2_{y.x}/n + \hat{S}_{b_1} (D^* - \hat{\bar{D}})^2)\ (1 - n/N)\ (1/L^{*2})$$

The sums of squares and crossproducts used in the "combined" and "overall" estimators were appropriately weighted by the inverse of the within-day error variances.

Monte Carlo estimates were found for the expected values, biases and variances of estimated performance and sample variance (Table 6), for both the "combined" and "overall" estimators. By comparing the results of the Monte Carlo trials (Table 6) with the example in Table 5, it will be noted that the random sample variance estimators are negatively biased in both instances. While the negative bias may be coincidental, the sources of bias present in the first example are probably also present in the estimates of performance using serial samples taken from the skidding population. These sources are the biased estimate of population variance obtained by using serial samples in a heterogeneous population, and the omission of a term to adjust for intercluster correlation.

The bias in the variance estimates becomes small as the size of the sample increases, or correspondingly, as the sampling fraction approaches one (Table 6). Unfortunately we cannot determine from these data whether the sample size or sample fraction is the more important cause of the reduction in sample variance.

Table 6. Results of Monte Carlo trials.

Sample Size	Performance Estimate				Variance Estimate		
	Expected Value	Variance	Sample Variance	Bias	Expected Value	Variance	Bias
a. Combined-Group Estimates							
50	11.44	1.6537	.0011	-.57	.4950	.000161	-1.1587
75	11.46	.8045	.0003	-.55	.2684	.000020	- .5362
100	11.46	.4753	.0007	-.56	.1781	.000005	- .2972
125	11.53	.2340	.0005	-.49	.1114	.000001	- .1227
150	11.57	.1420	.0000	-.45	.0702	.000000	- .0718
175	11.61	.0986	.0003	-.40	.0459	.000000	- .0527
200	11.64	.0563	.0001	-.38	.0289	.000000	- .0274
295	11.67			-.34	.0522[a]		
b. Overall-Group Estimates							
50	11.44	1.4409	.0006	-.60	.3448	.000004	-1.0961
75	11.54	.6326	.0003	-.48	.1915	.000005	- .4412
100	11.55	.3464	.0005	-.47	.1287	.000002	- .2177
125	11.60	.2010	.0003	-.42	.0849	.000000	- .1161
150	11.63	.1702	.0000	-.38	.0559	.000000	- .1135
175	11.65	.1410	.0002	-.36	.0385	.000000	- .1025
200	11.65	.0933	.0001	-.36	.0275	.000000	- .0676
295	11.65			-.36	.0514[a]		

[a] Variance estimates calculated without the finite population correction factor.

D. Comparison of Gross-Data and Time-Study Estimators

The performance estimators used with time-study and gross-data methods differ in the use of single turns and aggregated data for observation units and in their respective weighting procedures. More precise estimates are obtained with the time-study method by weighting each daily set of data by the inverse of its residual mean-square (Quenouille 1952, p. 130). This was done for the analyses described in the current and preceding chapters. Gross-data observations cannot be weighted in this manner because the residual mean-square associated with each day's performance can't be estimated. As a reasonable, though less satisfactory alternative, the sums of squares and crossproducts for each day's observations were weighted by the number of turns made in that day (Quenouille 1952, p. 116-117). Thus, performance estimates made using gross-data techniques may be less precise than time-study estimates based on the same sampling fraction because the weighting procedure is less exact, and the time-distance relationship is less precisely estimated; however, other factors may also influence the efficiency of these methods. What then is the efficiency of the two estimation methods assuming equal sampling fractions and ignoring sampling costs for the moment?

To provide an answer to this question, the gross-data method was investigated. The procedure used was similar to that used for the time-study estimates, and the same population of turns was used. However, the gross-data analyses differed from the time-study analyses in two ways; first, each of the eight daily sets of turns were aggregated and these aggregates were used as observation units, and second, the number of daily aggregates was small enough to obtain performance and variance estimates for the entire set of eight serial samples that could possibly be drawn from this population. Thus, the expected values of the estimators were exactly determined in the gross-data analyses.

Analyses were made using sampling fractions of .250, .375, .500 .625, .750 and 1.000. The actual variance for each set of performance estimates was calculated according to the formula

$$\hat{S}^2_{\hat{p}} = \sum_{i=1}^{8} (\hat{P}_i - \bar{P})^2/8$$

$$\text{where} \quad \bar{P} = \sum_{i=1}^{8} \hat{P}_i/8$$

Table 7. Efficiency of estimation methods.

Sampling Fraction	Variance Estimates				Efficiency[c]	
	Time-Study[a]		Gross-Data			
	True[e] Variance	Expected Value[a] of Estimator	True[f] Variance	Expected Value[b] of Estimator	Monte Carlo Estimate	Expected Value of Estimated Efficiency
.375	.362	.149	1.05	1.73	2.90	11.61
.500	.153	.080	.85	.68	5.56	8.50
.625	.077	.036	.61	.31	7.92	8.61
.750	.041	.020	.48	.16	11.71	8.00
1	--	.052	--	.52	--	10.00

See following page for footnotes.

Table 7 continued; footnotes.

[a]Based on 80 Monte Carlo samples using the following equation

$$\hat{S}^2_{\hat{p}_w} = (S^2_{y.x}/n + S^2_{bw}(D^* - \bar{D})^2)(1 - n/N)(1/L^{*2})$$

for each estimate. The expected value is

$$\hat{E}(S^2_{\hat{p}_w}) = \sum_{i}^{80} S^2_{\hat{p}_{wi}}/80$$

and n = number of turns in the sample.

[b]Based on the average of all possible variance estimates using

$$\hat{S}^2_{\hat{p}_b} = \hat{S}^2_{y.x_b}(1/n' + n(D^* - \bar{D})^2/n'[wD^2])(1 - n/N)(1/L^{*2})$$

where n' = number of days in the sample

and $$\hat{S}^2_{y.x_b} = n'\Sigma n_i(\hat{T}_i - T_i)^2/n(n' - 2)$$

[c]Efficiency (Time-study vs. Gross-data) = $S^2_{\hat{p}_b}/S^2_{\hat{p}_w}$

[d]Using the "combined" estimator.

[e]Based on Monte Carlo Estimator

$$S^2_{\hat{p}} = \sum_{i}^{80}(\hat{P}_i - \bar{P})^2/79$$

where $$\bar{P} = \sum_{i}^{80}\hat{P}_i/80.$$

[f]Exact determination based on all possible samples that could be taken in the population.

and compared to the mean of the corresponding set of random sample variance estimates

$$\bar{S}^2_{\hat{p}} = \sum_{i}^{8} S^2_{\hat{p}i}/8$$

Table 7 lists the estimates of the true sample variance and the expected value of estimated variance for the time-study and gross-data methods as well as the estimates of the true and estimated efficiencies for various sampling fractions. It will be noted that the true and estimated variances for gross-data estimates differ considerably. The peculiarities of the particular set of data used in these investigations and the small sample of daily performances available both contribute to this difference. Analyses of covariance do not support the null hypothesis that the expected value of the estimated variance is the same as the true sample variance for given sampling fractions, for either the gross-data or the time-study estimators.

VIII. ESTIMATION OF INVERSE AND SQUARED TERMS

A. Introduction

Even though a prediction model may be constructed that estimates performance satisfactorily, the performance estimate obtained from such an equation may still be biased. In addition to inherent bias due to non-linearity in the data, bias may be introduced through improper determination of the future, or expected, values of the independent variables used with the equation. In fact the estimate can also be biased if the dependent variable is improperly defined, even though the regression equation best describes the relationship in the population. The inverse of an estimate, e.g. $(1/\hat{L})$, is frequently used to estimate the expectation of the inverse of a random variable, e.g. $E(1/L)$. Analogously the square of an estimate, e.g. $(\hat{D}^2)$, is often used to estimate the expected value of a random variable that is a squared term, e.g. $E(D^2)$. The relative importance of these sources of bias in estimating skidding performance is now investigated and less biased procedures are developed and proposed. This problem does not arise if inverse or squared terms can be observed in a sample and the means of such terms can be calculated directly. Sometimes, because of the measurement or estimation procedure used, only an estimate of the mean and variance of the variable are available.

B. Unbiased Estimates of Inverse and Squared Terms

In estimating skidding performance, problems arise where estimates of the future value of the inverse of load size is needed, where a distance squared (D^2) term appears to be a significant variable in the performance equation under consideration, or where a regression equation is used to estimate "volume per unit time", $(P)^{-1}$, rather than its inverse (P), in the population of uniform time intervals.

An unbiased estimate of the expected value of a squared variable, e.g., distance (D^2), is derived from the definition of variance:

$$V(D) = E(D^2) - (E(D))^2$$

$$\text{Then} \quad E(D^2) = V(D) + (E(D))^2$$

$$= V(D) + \bar{D}^2$$

A sample based estimate of $E(D^2)$ can be obtained by using sample based estimates of V(D) and $\bar{D}$. Thus, the use of $\hat{\bar{D}}^2$ to estimate $E(D^2)$ results in a bias equal to -V(D).

If skidding distances are uniformly distributed, as they could be in some irregularly shaped skidding areas, an unbiased estimate of $E(D^2)$ is:

$$\hat{E}(D^2) = D^{*2}/3$$

where D* is the maximum skidding distance encountered in the logging area. This relationship is true for uniform distributions since $\bar{D}^2 = D^{*2}/4$ and $V(D) = D^{*2}/12$.

The problem of estimating the expected value of the inverse of a random variable has appeared frequently in the literature, e.g. Murthy and Pillai (1966). One procedure is to take the expectation of the Taylor's expansion of the function. When this was done for 1/L for the first two terms, the following approximation was obtained:

$$E(1/L) = 1/\bar{L} + V(L)/\bar{L}^3$$

The expectation of (1/L) can therefore be estimated using the sample based estimates of $\overline{L}$ and V(L) in this approximation.

A better approximation of E(1/L) is[a]

$$E(1/L) = 1/\overline{L} + 1.2(V(L)/\overline{L}^3 + 3(V(L)^2/\overline{L}^5)$$

None of these approximations will be satisfactory if $\overline{L}^3$ is less than V(L).

C. Empirical Studies

The bias associated with the foregoing estimators of the expected values of the inverse of a random variable, E(1/L), was investigated (Appendix D). Then studies were made of the bias contributed to the performance estimate by the various methods of estimating the expected values of squares and inverses (Appendix D) using three of the performance populations described in Chapter VI.

D. Observations and Conclusions

The use of $\hat{\overline{D}}^2$ instead of $\hat{\overline{D}}^2$ + V(D) caused the greatest bias. On the other hand, the use of $1/\hat{\overline{L}}$ (inverse of average load size in cunits) contributed the least bias, probably because the sample variance for estimated load size was very small and $\overline{L}$ was much larger than unity, i.e., 1.78 cunits.

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[a]This estimator was derived by T. A. Max, a graduate student in the Forestry Department of Iowa State University, using the first three terms of the Taylor's series and evaluating the remainder term.

The bias unaccounted for indicates that the values predicted by the regression equations were positively biased by from two to three per cent. This may be due to a non-linear component not accounted for by the regression estimators, or due to other unidentified sources of bias.

It appears that the suggested estimation procedures to reduce but by no means eliminate bias. All the biases were negative; the use of $\hat{\bar{D}}^2$ resulted in a bias of -5%, $1/\hat{\bar{L}}$ resulted in a bias of -.1% and $1/\hat{\bar{L}}_2$ resulted in a bias of -2.5%, where $\hat{\bar{L}}_2$ is cunits per hour.

While the suggested procedures are effective in reducing bias, it is probably better to select models in such a way that these problems do not arise.

IX. CONCLUSIONS AND PRACTICAL APPLICATIONS

A. Limitations of the Study

The preceding chapters described investigations made to better understand how to estimate skidding performance in a harvesting operation. These investigations used performance data from a heterogeneous population of some 614 machine turns. The results of these investigations are summarized and their applicability to time-study and gross-data techniques are described. Further studies needed are also indicated.

Inferences made from these studies are weakened by the repeated use of the data for the various analyses. Once a model was selected, there was no opportunity to test its validity with different data. The data used to evaluate the sampling properties of the time-study and gross-data methods were obtained by combining the large-skidder data for three companies. These companies had different objectives as well as different management and labour practices. Erroneous conclusions could easily have been caused by such data.

Do these results apply to other companies and machine types? The studies used a very small sample taken from three different companies' operations. Inference that the same model characteristics are more broadly applicable is therefore limited.

Nevertheless the estimation properties examined here may also be applicable to a broad range of felling, forwarding, slashing and hauling elements which are subject to the same operational, management and environmental factors as timber skidding. However, more studies should be made to confirm this broader application.

B. Findings of the Investigations

Good estimates of performance are needed to set fair contract prices and to make accurate cost appraisals of harvesting operations. Expensive losses in terms of money, man-, and machine-hours can result from poor estimates. While studies have been made to identify factors affecting performance in an effort to improve performance estimation, very little has been done to assess the measure of precision associated with performance estimators. Precision must be measured so that the decision-maker can select the most efficient method. However the problem of measuring precision of performance estimates is complicated by the fact that serial observations are obtained by time-study or gross-data gathering techniques used to make these estimates. Thus, the random sample variance estimator which is the common measure of precision does not perform well in these types of data.

These investigations were made to find ways of improving (and measuring) the precision of time-study and gross-data methods for estimating performance.

We found that precision could be improved by using weighted regression techniques, where the weighting factor for time-study estimates was the inverse of the residual mean square for each daily set of observations, and the weighting factor for the gross-data methods was the number of turns per day.

Both the time-study and gross-data methods estimated the same time-distance relationship and this relationship was the same for the various machine types and companies included in the investigations. If costs were ignored, the time-study method estimated performance five to ten times more efficiently than the gross-data method. However, this efficiency could be offset by the higher cost of time-study procedures compared to the cost of gross-data methods.

A bias of up to five per cent could be introduced in the performance estimate by the improper estimation of the future levels of the independent variables or by improper specification of the dependent variable. In particular, bias was introduced if the expected values of the square of distance $E(D^2)$, or the inverse of load size $E(1/L)$ were estimated by the square of mean distance $(\hat{\bar{D}}^2)$, or the inverse of estimated mean load $(1/\hat{\bar{L}})$ respectively. Similarly bias could be introduced if performance, measured as "hours per cunit", were estimated by taking the inverse of a performance estimate in which performance had been expressed as "cunits per hour". In addition to assessing the magnitude of these biases the investigations showed that much of this bias could be eliminated if suggested procedures were followed.

C. Practical Applications of These Results

We can use the results of this study to choose between the time-study and gross-data methods for estimating skidding performance. In particular, the results should help to decide approximately how large time-study or gross-data sample must be taken to achieve an estimate of specified precision, and also to decide whether to use the time-study or gross-data method to achieve an estimate of specified precision at minimum cost.

In the next few paragraphs the following notation and abbreviations will be used.

$$S^2_{\hat{p}w} = (1/n)\ (S^2_{y.x} + S^2_{y.x_w}(D^* - \bar{D})^2/S^2_D)/L^{*2} \tag{1}$$

$$= (1/n)\ [A] \tag{2}$$

where all notation is as previously defined except S^2_D which is the weighted within-day variance of skidding distances estimated by

$$\hat{S}^2_D = \sum_i^{n'} \sum_j^{n_i} w_i (D_{ij} - \bar{D}_i)^2/n\ (\Sigma w_i) \tag{3}$$

and
$$S^2_{\hat{p}b} = (1/n')\ S^2_{y.\ b}(1 + (D^* - \bar{D})^2/S^2_{Db})/L^{*2} \tag{4}$$

$$= (1/n')\ [B] \tag{5}$$

Where S^2_{Db} is the weighted between-day variance of skidding distances, n' is the number of days included in the gross-data survey, and W_i is the inverse of the within-day sample variance $(1/S^2_{y.xi})$.

It is assumed that the quantities [A] and [B] are known prior to the survey.

The time-study method will be used in preference to the gross-data method if, for specified precision,

$$[B]/[A] > c_t/c_d \tag{6}$$

where c_d is the cost of making a daily observation for the gross-data method and c_t is the cost of making a turn observation for the time-study method.

This is because, given $S^2_{\hat{p}w} = S^2_{\hat{p}b}$, the time-study method is preferred if

$nc_t < n'c_d$

or $\quad n'/n > c_t/c_d$

but $\quad [A]/n = [B]/n'$

therefore $\quad n'/n = [B]/[A]$

so that $\quad [B]/[A] > c_t/c_d \quad$ Q.E.D.

If observations, made over an 80 day period with the gross-data method, gave an estimate with the same precision ($S_{\hat{p}}^2 = .051$) as 295 observations using the time-study method, and it cost \$1.50 to make either a time-study observation or a gross-data observation, the gross-data technique would obviously be preferred since the total cost for the time-study estimate would be \$442.50 compared to \$120.00 for the gross-data method.

In general the gross-data technique would be preferred. Equations (1) and (4) suggest that the exception would be if considerable precision were gained in estimating the time-distance relationship (b_1) using the time-study method and if the $(D^* - \overline{D})^2$ component was relatively large, or if between-day differences in performance were very large.

The time-study method may also be preferred if the performance estimate were needed immediately.

To determine the sample size for each method to achieve specified precision (S_p^{2*}), the following approximations may be used:

$$n' = [B]/S_p^{2*}$$

$$\text{and} \quad n = [A]/S_p^{2*}$$

D. Further Studies Indicated

More studies are needed, to test the inferences made in this study, and to provide a better understanding of the relationships between harvesting performance and the relevant independent variables.

The studies needed most are those which can produce better prediction equations.

Important unexplained between-day differences exist which require the use of special regression techniques for heterogeneous data in order to achieve better precision. The factors causing these between-day differences should be identified, quantified, and their relationship incorporated into the prediction equations. Some factors have already been tentatively identified such as the number of turns made in a given area and the operator identification number. Other factors include crew aggressiveness, day of the week, and local site conditions resulting from day-to-day changes in the weather. Much work has been done already, e.g. McCraw (1967) and Bennett, Winer and Bartholomew (1967); however, more work should be done with particular emphasis on the prediction of next year's performance using concomitant variables whose future values can either be currently observed or predicted.

Studies are needed to determine which time-study estimator is preferable, the estimator based on "combined" or "overall" weighted sums of squares and crossproducts. The observations made in this study are inconclusive, although the "overall" estimator was more precise and less biased.

Studies are also needed to determine how the presence of time-study personnel influences performance, and whether this influence can be predicted and corrected or not. "Levelling" techniques have been developed to correct for this bias but improvements are indicated (Killander, 1960).

Other studies are indicated to develop performance equations for other elements in the timber harvesting operation. Many modifications in wheel skidding have been made since 1962 when the data used in these studies was gathered, and new multiprocessors have been developed. Some of the relationships observed in this study may also apply to these newer harvesting techniques; however, verification is needed. Finally, only the prediction of performance for individual elements in a harvesting operation have been investigated here. Studies are also needed to determine how the interplay among various combinations of harvesting elements affects the performance of any single element.

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XI. ACKNOWLEDGMENTS

I wish to acknowledge with sincere thanks the effort expended by the various members of my Ph.D. committee in terms of patience, understanding, time and constructive criticism which they have offered in the course of this research.

Thanks to Dr. Paul Hinz and Mr. Bud Meador of the Statistical Service Department, Iowa State University for their help in the initial programming and analysis of data.

Thanks to Mr. W. E. McCraw who graciously supplied the data for this study while with the Forest Management Institute, Department of Fisheries and Forestry, Canada.

Acknowledgment with special thanks for the consideration given by my various supervisors, Dr. Carl Stoltenberg and Dr. Henry Webster of Iowa State University and Dean J. W. B. Sisam at the University of Toronto who permitted me to coordinate my various teaching responsibilities with my research.

My thanks to Mrs. K. Olson and Miss M. Grasley for their typing and editing services.

Thanks to my wife and family for their patience and understanding in the face of my frustrations.

XII. APPENDIX A

A. Procedure Followed for Model Fitting in Chapter VI

The full model fitted to each set of daily data was derived from physical relationships thought to exist among time, load and distance. It was hypothesized that turn-time consists of travel-time, load-handling time and delay time, and that travel time was related directly to one-way skidding distance (D) and distance squared (D^2). Load handling time was related to load size (L) and delays were related in a complex manner to distance, load size and unidentified factors. Thus, turn time (T) in minutes per turn could be expressed as follows:

$$T_i = b_o' + b_1'L_i + b_2'D_i + b_3'D_i^2 + b_4'D_iL_i + E_i,\ i = 1, 2, \ldots, N.$$

where E_i was an error term with expected value = 0 and constant variance for all i and b_o', ..., b_4' are constants. Thus, the performance ratio (minutes per cunit) could be estimated for each load by dividing the above expression by L_i, so that

$$T_i/L_i = b_o + b_1(1/L_i) + b_2(D_i/L_i) + b_3(D_i) + b_4(D_i^2/L_i)$$

This was the full model that was fitted to the data in each area. The reduced models fitted were:

1) $(T_i/L_i) = b_o$

2) $(T_i/L_i) = b_1(1/L_i) + b_2(D_i/L_i)$

3) $(T_i/L_i) = b_o + b_1(1/L_i) + b_2(D_i/L_i)$

4) $(T_i/L_i) = b_o + b_1(1/L_i) + b_2(D_i/L_i) + b_3(D_i)$

5) $(T_i/L_i) = b_o + b_1(1/L_i) + b_2(D_i/L_i) + b_3(D_i) + b_4(D_i^2/L_i)$

Table 8 summarizes the results of these studies.

The initial step in analysing the fitted equations was to see if the residual errors were autocorrelated. If so, the prediction equations could be modified to produce more efficient estimates.

Autocorrelation exists if the error terms in the fitted equation are in some way related over serial observations. If this condition exists, then the regression coefficients will be unbiasedly estimated, but with needlessly large sampling variance. Also, by using least-squares formulae, the estimated sampling variances of the regression coefficients will be too small, and the predictions made with such formulae will have needlessly large sampling variances (Johnston, 1963).

There are several factors in skidding that result in autocorrelated errors. Performance on successive turns from a given area will usually improve as trail conditions improve and travel speeds increase. Groups of machines may exhibit autocorrelated error patterns as high and low producers' performance is logged in arrival sequence at the landing. Errors can also be related through daily, weekly and seasonal trends in worker behaviour and environmental changes.

Residual errors were plotted in sequence and examined using the full model fitted to data for two different days. The Durbin-Watson statistic (Theil and Nagar, 1961) was also calculated. The residual errors associated with the equations showed a roughly cyclical pattern presumably related to the various skidding units, however the period of the cycle was irregular.

The Durbin-Watson statistic gave non-significant values for the model tested in both areas, i.e. 2.20 and 1.95.

It was concluded that the data were not strongly autocorrelated, but that more precise estimates might have been obtained had the skidding unit number and the number of turns made from a given work site been included in the prediction equation. Since these variables were not recorded, they could not be included in the models studied here.

The residual errors, when plotted, revealed a positively skewed distribution. This error distribution is characteristic of most operator dependent performances. It was concluded, however, that the skewness was not sufficient to warrant a logarithmic transformation of the dependent variable. The logarithmic transformation implies a relationship between performance, distance and load size that is contrary to the actual physical relationship, and its use was not adopted for this reason.

The best of the models was then selected from among the five models summarized in Table 8. The criteria for selection was a consistently small residual mean square and a small number of terms in the equation. Of the five equations listed in Table 8, model number two was selected.

$$P = b_1(1/L) + b_2(D/L)$$

The estimates of the regression coefficients and their standard errors, as well as the standard error for this equation are listed for each set of daily data in Table 9.

Tables 8 and 9 indicate that a high proportion of the variation remains unexplained, and that for half of the sets of data, it is

Table 8. Residual mean squares for models using various daily sets of data

Day No.	Co. No.	Mach. Type	No. Obs.	Residual Mean Square				
				Model Number[a]				
				1	2	3	4	5
1	1	L	28	23.77	15.82	16.38	13.19	13.18
2	1	L	47	14.18	7.64	6.90	6.53	5.69
3	1	L	11	13.79	10.57	10.96	7.62	7.36
4	1	L	34	7.23	7.00	7.47	7.00	6.62
5	1	M	19	77.37	67.90	68.68	69.76	74.51
6	1	L	18	23.52	5.19	4.35	4.60	4.95
7	2	M	17	84.13	53.89	55.11	55.08	46.43
8	2	M	89	171.42	130.16	130.46	130.83	131.52
9	2	M	70	30.47	18.19	17.26	17.51	17.47
10	2	M	40	138.58	96.42	95.63	98.28	95.03
11	2	M	55	263.08	237.04	240.33	243.39	245.52
12	2	L	23	68.27	82.40	60.19	77.27	75.67
13	2	M	6	82.15	30.33	40.00	59.24	62.78
14	3	M	23	146.48	139.10	145.89	152.23	159.52
15	3	L	61	68.96	19.76	20.07	20.40	20.03
16	3	L	73	15.89	8.37	8.37	8.48	8.54

[a]Model No. 1 $P = b_o$

2 $P = b_1(1/L) + b_2(D/L)$

3 $P = b_o + b_1(1/L) + b_2(D/L)$

4 $P = b_o + b_1(1/L) + b_2(D/L) + b_3(D)$

5 $P = b_o + b_1(1/L) + b_2(D/L) + b_3(D) + b_4(D^2/L)$

difficult to decide (using t-tests) whether the relationship between distance and performance is significant or not. Also, variation exists among the standard errors displayed in Table 9.

These observations indicate the need for the analyses of heterogeneous data as described in Appendix B.

Table 9. Coefficients and standard errors for the performance equation[a].

Day No.	Co. No.	Mach. Type	No. Obs.	1/L		D/L		Stand. Error
				b_1	SE $_{b1}$	b_2	SE $_{b2}$	
1	1	L	28	8.83	4.68	1.01	.44	3.98
2	1	L	47	6.99	1.94	.80	.17	2.76
3	1	L	11	40.90	12.44	-7.33	3.65	3.25
4	1	L	34	14.47	2.54	.17	.25	2.65
5	1	M	19	19.32	4.42	-.49	.96	8.24
6	1	L	18	11.98	9.40	.42	.70	2.28
7	2	M	17	13.20	3.25	.72	.73	7.34
8	2	M	89	14.98	4.37	-.20	.37	11.41
9	2	M	70	-2.33	2.03	.85	.12	4.25
10	2	M	40	3.42	21.86	.82	1.55	9.82
11	2	M	55	16.39	2.88	.71	.22	15.40
12	2	L	23	23.42	16.45	.50	.60	9.08
13	2	M	6	-21.50	87.55	8.65	3.04	5.51
14	3	M	23	48.02	21.41	-.13	.63	11.79
15	3	L	61	14.23	1.93	.51	.12	4.44
16	3	L	73	8.48	2.30	.63	.11	2.89

[a]Model fitted is $P = b_1(1/L) + b_2(D/L)$.

Table 10. Coefficients and standard errors for turn-time equation[a].

Day No.	Co. No.	Mach. Type	No. Obs.	Intercept		Distance (D)		Stand. Error
				b_o	SE_{bo}	b_i	SE_{b1}	
1	1	L	28	13.55	4.40	.55	.38	6.66
2	1	L	47	6.21	2.26	.96	.19	5.11
3	1	L	11	37.80	10.98	-6.26	3.21	5.61
4	1	L	34	15.47	3.15	.14	.29	6.08
5	1	M	19	17.99	15.60	-.11	1.08	5.19
6	1	L	18	7.65	6.34	.87	.51	2.97
7	2	M	17	13.97	3.06	.49	.68	4.06
8	2	M	89	17.51	3.86	-.20	.32	7.30
9	2	M	70	-2.38	2.08	.87	.12	2.86
10	2	M	40	-.64	26.51	1.17	1.87	7.15
11	2	M	55	17.54	3.17	.64	.22	10.38
12	2	L	23	24.42	16.84	.49	.61	12.28
13	2	M	6	-217.93	92.08	8.75	3.20	4.56
14	3	M	23	49.29	19.94	-.16	.58	16.14
15	3	L	61	13.90	2.26	.52	.13	4.27
16	3	L	73	7.79	2.28	.65	.10	5.48

[a]Model fitted is $T = b_o + b_1 D$

XIII. APPENDIX B

A. Analyses of Heterogeneous Data

After finding the performance equation that best fits the sixteen daily sets of data, it was noted that the standard error varied considerably from one set of data to the next (Appendix A). Therefore, the heterogeneity of the data was analysed.

The analyses were made in the following order. First, Bartlett's tests (Snedecor and Cochran 1968, p. 65) were made to test for differences among the residual mean-squares for various company, machine-type combinations of the daily data in order to decide whether a weighted regression should be used or not. Then analyses were made to test for critical differences between both the regression coefficients and the intercept terms estimated for the various combinations of data.

The following model was used in these analyses:

$$T_1 = b_o + b_1 D$$

where T_1 and D are turn time (minutes) and skidding distance (hundreds of feet), respectively, for machine turns. In addition to being a simpler model to use, this model is the one that would be of particular interest in time-study work. Performance would be estimated by multiplying predicted mean turn-time ($\hat{T}$) by the predicted mean inverse of load ($\widehat{1/L^*}$).

The model is compatible with the model developed in Appendix A, and both models exhibit virtually the same error patterns. The Durbin-Watson statistics were 2.20 and 1.95 for the data tested in

Appendix A, while the Durbin-Watson statistics were 2.24 and 1.96, respectively, for the same data using the model described in this appendix. A comparison of the standard errors obtained by fitting the two equations to the sixteen sets of daily data indicated that the standard errors were comparable (Tables 9 and 10). This observation was confirmed by the use of Bartlett's tests which were subsequently carried out.

Table 11 summarizes the results of the tests of homogeneity of variance for various combinations of daily data.

Only the five company-one, large-skidder groups of data may have had homogeneous variances. It was concluded that the residual mean squares for the various data groupings were sufficiently different to warrant using weighted regressions in the remainder of the analyses. Observations were therefore weighted by the inverse of the residual mean square for the day in which the observations were made.

Analyses of covariance were made to test for differences among regression coefficients and intercept terms (Williams 1959, p. 129-137) using weighted sums of squares and crossproducts in the various groupings of data. The results are summarized in Tables 12, 13 and 14 for large- and medium-skidder data groupings and for the three company groupings. It will be noted that the F-tests indicated marginal differences between coefficients fitted to the large-skidder data for company number one, and for the medium-skidder data for company two. As these groups of data were combined with more groups, the F-values tended to decrease. The conclusion was that the time-distance relationship can be satisfied using a common regression coefficient

Table 11. Tests of homogeneity of variance for various combinations of daily data.

Equation: $T = b_o + b_1D$

Daily Groups Compared	No. of Groups	No. of Obs'ns	Deg. of Freedom	Bartlett[a] X^2
Co. 1-large skidders	5	138	4	10.8 **
Co. 1 and 3-large skidders	7	272	6	19.5 **
All large skidders	8	295	7	62.8 **
Co. 2-medium skidders	6	277	5	94.0 **
All medium skidders	8	319	7	140.2 **
All days	16	614	15	217.5 **
All company 1	6	157	5	11.7 **
All company 2	7	300	6	109.9 **
All company 3	3	157	2	82.2 **

[a]ns - not significant at 5% probability level

* - significant at 5% probability level

** - significant at 1% probability level

for all company-machine-type groupings. In other words, the turn-distance relationship appeared to be independent from company and machine type influences in these data.

Table 12. Analysis of heterogeneous data for large skidders.

Equation: $T = b_o + b_1 D$

	$[wD^2]$	b_1	SE b_1	Deviations from Regression			
				d.f.	SS	MS	F^a
Company 1-large skidders							
Separate est.-b_1				128	128.000	1.000	
Combined est.-b_1	50.96	.698	.144	132	138.519	1.049	
diff. due to b_1				4	10.519	2.630	2.63*
Overall est.-b_1	79.61	.533	.116	136	146.794	1.079	
diff. due to intercept				4	8.275	2.069	1.97^{ns}
Company 1 and 3-large skidders							
Separate est.-b_1				258	258.000	1.000	
Combined est.-b_1	207.44	.625	.070	264	269.544	1.021	
diff. due to b_1				6	11.544	1.924	1.92^{ns}
Overall est.-b_1	435.78	.512	.050	270	291.692	1.080	
diff. due to intercept				6	22.148	3.691	3.61*
All large skidders							
Separate est.-b_1				279	279.000	1.000	
Combined est.-b_1	210.18	.624	.070	286	290.592	1.016	
diff. due to b_1				7	11.592	1.656	1.66^{ns}
Overall est.-b_1	461.66	.560	.049	293	326.196	1.113	
diff. due to intercept				7	35.604	5.086	5.01*

[a] ns - not significant at 5% level

* - significant at 5% level

** - significant at 1% level

Table 13. Analysis of heterogeneous data for medium skidders.

Equation: $T = b_0 + b_1 D$

	$[wD^2]$	b_1	SE b_1	Deviations from Regression			
				d.f.	SS	MS	F^a
Company 2-medium skidders							
Separate est.-b_1				265	265.000	1.000	
Combined est.-b_1	98.65	.717	.103	270	281.108	1.041	
diff. due to b_1				5	16.108	3.222	3.22*
Overall est.-b_1	311.65	.155	.080	275	538.380	1.958	
diff. due to intercept				5	257.272	51.454	49.42**
All medium skidders							
Separate est.-b_1				303	303.000	1.000	
Combined est.-b_1	102.39	.685	.101	310	321.850	1.038	
diff. due to b_1				7	18.850	2.693	2.69*
Overall est.-b_1	419.65	.180	.070	317	652.036	2.057	
diff. due to intercept				7	330.186	47.170	45.43**
All days							
Separate est.-b_1				582	582.000	1.000	
Combined est.-b_1	312.57	.644	.057	597	612.698	1.026	
diff. due to b_1				15	30.698	2.047	2.05*
Overall est.-b_1	883.28	.429	.047	612	1200.680	1.962	
diff. due to intercept				15	587.982	39.199	38.19**

[a]ns - not significant at 5% level

* - significant at 5% level

** - significant at 1% level

Table 14. Analysis of heterogeneous data by companies.

Equation: $T = b_o + b_1D$

	$[wD^2]$	b_1	SE b_1	Deviations from Regression d.f.	SS	MS	F^a
Company 1							
Separate est.-b_1				145	145.000	1.000	
Combined est.-b_1	51.77	.685	.142	150	156.037	1.040	
diff. due to b_1				5	11.037	2.207	2.21^{ns}
Overall est.-b_1	108.09	.400	.101	155	168.644	1.088	
diff. due to intercept				5	12.607	2.521	2.42*
Company 2							
Separate est.-b_1				286	286.000	1.000	
Combined est.-b_1	101.39	.711	.101	292	302.241	1.035	
diff. due to b_1				6	16.241	2.707	2.71*
Overall est.-b_1	673.86	.282	.055	298	612.697	2.056	
diff. due to intercept				6	310.456	51.743	49.99**
Company 3							
Separate est.-b_1				151	151.000	1.000	
Combined est.-b_1	159.41	.587	.079	153	153.373	1.002	
diff. due to b_1				2	2.373	1.187	1.19^{ns}
Overall est.-b_1	208.33	.493	.075	155	180.104	1.162	
diff. due to intercept				2	26.731	13.365	13.33**

[a]ns - not significant at 5% level

* - significant at 5% level

** - significant at 1% level

XIV. APPENDIX C.

A. Efficiency of Regression Coefficient Estimates Using Time-Study and Gross-Data Techniques.

It was shown in Appendix B that the sixteen estimates of the regression coefficient (b_1), obtained by using the daily sets of data separately, were probably estimating a common time-distance relationship. However, in Appendix B the estimates of b_1 were based on either "combined" or "overall" weighted sums of squares and crossproducts derived from observations taken within each day separately. These are referred to as within-day estimates and are easily obtained using time-study data. A second method of estimating the regression coefficient (b_1) uses "between-day" sums of squares and crossproducts according to the method described by Williams (1959, p. 156-158). This estimation method is the one best suited for use with gross-data. The efficiency of these two techniques for estimating the regression coefficient (b_1) is now investigated.

Regression coefficients and their sample variances were first calculated from data for each company, machine-type combination using within- and between-day weighted sums of squares and crossproducts. T-tests were then made to compare the two estimates of the time-distance regression coefficients. Finally the efficiencies of the within-day estimates compared to the between-day estimates were determined for each data combination and subsequently used as weighting factors for pooled between-within-day estimates of the regression coefficients.

The results are listed in Tables 15, 16, and 17, and, with the exception of the large-skidder data for company-one where the difference is only marginally significant, the within- and between-day estimates appear to be estimating a common relationship. In other words the time-study and gross-data techniques both appear to estimate the same relationship between turn-time and distance.

The efficiency of the between-day estimates compared to the within-day estimates varied from 1.13 for company-one performances to .018 for company-two performances. Generally the within-day estimates were much more efficient in those sets of data where the intercepts were more heterogeneous.

The efficiency of the within-day estimates compared to the between-day estimates can be explained by the fact that the variance of between-day estimates includes variation from factors that vary within and between each day, while the within-day estimates are only subject to variation due to factors that vary within each day. Thus variance of between-day estimates should be larger.

The final analysis made in this series is the analysis of variance showing the variation accounted for using the combined within- and between-day estimates of the coefficients, differences from regression, and residual sums of squares. Again the procedure followed is that described by Williams (1959) for heterogeneous data. The analyses of variance are summarized in Tables 18, 19 and 20. The estimated coefficients are all significantly different from zero. As previously noted, marginally significant differences still exist between the between- and within-day estimates for company-one, large-skidder data. This difference is obscured as more days are included for analysis and it is concluded that the difference was probably due to chance variation.

Table 15. Comparison of within- and between-day estimates of regression coefficients-large skidders.

		b_1	d.f.	S^2_{b1}	SE b_1	t	Weight[a]
Company 1-Large Skidders							
Between		.182	3	.039	.198		
Within		.698	132	.020	.141		
Total	diff.	.516		.059	.244	2.119*	.930
Company 1 and 3-Large Skidders							
Between		.474	5	.018	.135		
Within		.625	264	.005	.068		
Total	diff.	.151		.023	.151	.999[ns]	.259
All Large Skidders							
Between		.558	6	.024	.156		
Within		.624	286	.005	.068		
Total	diff.	.066		.029	.170	.387[ns]	.175

ns - not significant at 5% level

* - significant at 5% level

** - significant at 1% level

[a]Weight - denotes proportion of between-day sum of squares added to within-day sum of squares for combined estimate of b_1.

Table 16. Comparison of within- and between-day estimates of regression coefficients-medium skidders.

		b_1	d.f.	$S^2_{b_1}$	SE b_1	t	Weight[a]
Company 2-Medium Skidders							
Between		.029	4	.263	.513		
Within		.717	270	.011	.104		
Total	diff.	.688		.274	.524	1.314^{ns}	.0185
All Medium Skidders							
Between		.102	6	.159	.399		
Within		.685	310	.101	.102		
Total	diff.	.583		.169	.412	1.415^{ns}	.021
All Days							
Between		.335	14	.073	.269		
Within		.644	597	.003	.058		
Total	diff.	.309		.076	.275	1.122^{ns}	.026

ns - not significant at 5% level

* - significant at 5% level

** - significant at 1% level

[a]Weight - denotes proportion of between-day sum of squares added to within-day sum of squares for combined estimate of b_1.

Table 17. Comparison of within- and between-day estimates of regression coefficients-companies.

		b_1	d.f.	S^2_{b1}	SE b_1	t	Weight[a]
Company 1							
Between		.110	4	.016	.128		
Within		.685	150	.021	.144		
Total	diff.	.575		.037	.193	2.985*	1.130
Company 2							
Between		.203	5	.247	.497		
Within		.711	292	.010	.102		
Total	diff.	.508		.257	.507	1.002^{ns}	.018
Company 3							
Between		.619	1	.736	.858		
Within		.587	153	.006	.080		
Total	diff.	.031		.742	.861	$.036^{ns}$	.038

ns - not significant at 5% level

* = significant at 5% level

** - significant at 1% level

[a]Weight - denotes proportion of between-day sum of squares added to within-day sum of squares for combined estimate of b_1.

Table 18. Analyses of variance: combined regressions for large skidder groups.

	d.f.	SS	MS	F	b	SE b
Company 1-large skidders						
Combined regression	1	21.027	21.027	20.05**		
Difference of regressions	1	4.665	4.665	4.44*		
Residual	135	141.669	1.049			
Total	137	167.361			.521	.117
Company 1 and 3-large skidders						
Combined regression	1	92.757	92.757	90.80**		
Difference of regressions	1	1.003	1.003	.98^{ns}		
Residual	269	274.649	1.021			
Total	271	368.409			.593	.062
All large skidders						
Combined regression	1	94.616	94.616	93.20**		
Difference of regressions	1	.181	.181	.178^{ns}		
Residual	292	296.701	1.016			
Total	294	391.498			.613	.063

ns - not significant at 5% level

* - significant at 5% level

** - significant at 1% level

Table 19. Analyses of variance: combined regressions for medium skidder groups.

	d.f.	SS	MS	F	b	SE b
Company 2-medium skidders						
Combined regression	1	48.832	48.832	46.80**		
Difference of regressions	1	1.816	1.816	1.74^{ns}		
Residual	274	285.261	1.041			
Total	276	335.909			.690	.101
All medium skidders						
Combined regression	1	45.963	45.963	44.25**		
Difference of regressions	1	2.090	2.090	2.01^{ns}		
Residual	316	328.071	1.038			
Total	318	376.124			.650	.098
All days						
Combined regression	1	129.900	129.900	126.50**		
Difference of regressions	1	2.603	2.603	2.54^{ns}		
Residual	611	626.031	1.025			
Total	613	758.534			.630	.056

ns - not significant at 5% level

* - significant at 5% level

** - significant at 1% level

Table 20. Analyses of variance: combined regressions for company groups.

	d.f.	SS	MS	F	b	SE b
Company 1						
Combined regression	1	15.630	15.630	15.02**		
Difference of regressions	1	9.456	9.456	9.09**		
Residual	154	160.191	1.040			
Total	156	185.277			.368	.097
Company 2						
Combined regression	1	50.293	50.293	46.20**		
Difference of regressions	1	1.129	1.129	1.091 ns		
Residual	297	307.425	1.035			
Total	299	358.847			.690	.099
Company 3						
Combined regression	1	55.583	55.583	55.50**		
Difference of regressions	1	.021	.021	.021 ns		
Residual	154	154.370	1.0024			
Total	156	209.973			.588	.079

ns - not significant at 5% level

* - significant at 5% level

** - significant at 1% level

XV. APPENDIX D

A. Studies of Bias in Performance Estimation

The following investigations were undertaken to determine the size of the bias incurred through some commonly used estimation procedures.

The first study was made to determine the bias introduced by estimating the expected value of the inverse of load size E(1/L), by different methods. Four normally distributed load-size populations with means of .67, 1.00, 1.33 and 1.66 cunits respectively, and coefficients of variation equal to .30, were used in this investigation.

Approximations of the true values E(1/L) were found by treating the normal distributions as discrete distributions containing seventy-five class intervals. The probability of observing loads in each size class was $P(L_i)$, i = 1, 2, ..., 75, where (L_i) is the i^{th} load size class. The expectation of (1/L) was found for each load-size distribution by the following equation:

$$E(1/L) = \sum_{i=1}^{75} P(L_i)(1/L_i)$$

The estimators $(1/\hat{L})$, $1/\hat{L} + V(L)/\hat{L}^3$ and Max's estimator, $1/\hat{L} + 1.2 (V(L)/\hat{L}^3 + 3(V(L)^2/\hat{L}^5)$, introduced in Chapter VIII, were used with Monte Carlo techniques to obtain estimates of E(1/L). One hundred and fifty load sizes were randomly generated in each trial. The Monte Carlo estimates of the expected values of the estimators were then compared with the true values to determine the bias (Table 21). None of the estimators performed well in the populations with means of less than one cunit where the variance was greater than load size cubed.

Table 21. Comparison of various estimators of $E(1/L)$.

Sample size =150

Coefficient of variation 30%

Mean Load	$E(1/L)$	$1/\hat{L}$	Bias	$1/\hat{L}+V(L)/\hat{L}^3$	Bias	Max Est.[a]	Bias
.67	1.6355	1.3358	-.2997	1.4551	-.1804	1.5154	-.1201
1.00	1.0951	.9851	-.1100	1.0816	-.0135	1.1219	+.0268
1.33	.8234	.7508	-.0626	.8133	-.0101	.8237	+.0003
1.66	.6645	.6040	-.0605	.6492	-.0153	.6735	+.0090

[a]Max Est. $= 1/\hat{L} + 1.2(V(L)/\hat{L}^3 + 3V(L)^2/\hat{L}^5)$

Next, a series of studies were conducted in three of the performance populations described in Chapter VI to examine the bias contributed to the performance estimate by various methods of estimating the expected values of squares and inverses. The true mean performance for these populations was known beforehand and the biases associated with the various estimators were estimated using Monte Carlo procedures.

Mean performance was estimated using all the observation units present in each of three different populations so that error in the regression estimate due to sampling was zero. Any bias would be due either to the fit of the equations or due to the methods of estimating the mean values of the independent variables, in particular those variables which included either inverse or squared terms. One hundred random estimates of mean load size were generated to produce 100 estimates of performance.

Table 22. Sources of bias in performance estimators.

Estimator Number[a]	Sample Size	Obs'n Unit Size	Monte Carlo Est.	Sources of Bias: Due to $\hat{D}^2$	Due to $1/\hat{L}$	Due to $\hat{D}/\hat{L}$	Due to $1/\hat{L}_2$	Bias Not Explained
1	295	1 turn	11.52	-.829	-.017	--	--	+.346
2	52	2 hrs.	10.96	--	--	-.005	-.431	-.624
3	65	8 cunits	12.55	--	--	-.009	--	+.539

[a]1. $P = (b_o + b_1\hat{D} + b_2\hat{D}^2)/\hat{L}$

2. $P = \bar{T}_2/(b_o + b_1\hat{D}/\hat{L} + b_2\bar{L}) = \bar{T}_2/\hat{L}_2$

3. $P = (b_o + b_1\hat{D}/\hat{L} + b_2\hat{L})/\bar{L}_3$

Mean load size estimates were generated by the standard IBM random numbers generating subroutine "Randu".

The performance equations used in these studies were modifications of the equations developed in Appendix A. The modifications were the inclusion of square and inverse terms as independent variables in the equations developed previously.

For the turn population the estimator used was:

$$\hat{\bar{P}} = (b_o + b_1\hat{\bar{D}} + b_2\hat{\bar{D}}^2)/L_1$$

The estimator for the population of uniform time intervals was developed to show the bias incurred by using the inverse of estimated volume per fixed time interval, $(1/\hat{L}_2)$, rather than estimating directly the inverse of volume per fixed time, $(1/L)$.

The performance equation used in this set of trials was:

$$\hat{\hat{P}} = \bar{T}_2/(b_o + b_1\hat{D}/\hat{L} + b_2\hat{L})$$

Finally the estimator used in the population of uniform production units was selected to show the influence of using the inverse of estimated load size. The estimator used was:

$$\hat{\hat{P}} = (b_o + b_1\hat{D}/\hat{L} + b_2\hat{L})/\bar{L}_3$$

It will be remembered that these unweighted regression will result in less efficient estimates than the techniques proposed in Chapter VII.

The Monte Carlo estimates of performance in the three populations were compared with estimators that were adjusted to correct for the use of $\hat{\bar{D}}^2$, $1/\hat{\bar{L}}$ and $1/\hat{\bar{L}}_2$.

The use of $\bar{D}^2$ contributed the greatest bias, while the use of $(1/\hat{\bar{L}})$ to estimate $(1/\bar{L})$ contributed the least bias in performance estimation (Table 22).